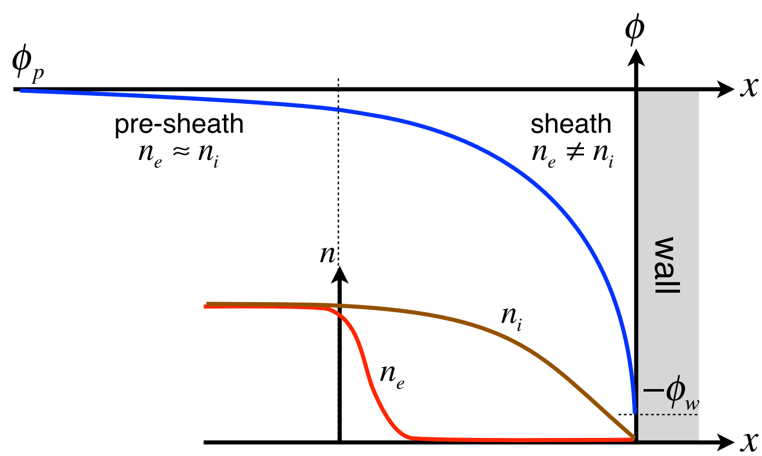
Lecture 11: Overview of the physics in the plasma sheath

Electrons in plasmas are much more mobile (higher temperature, lower mass) than ions, and consequently their fluxes are much greater. Because of this, materials in contact with plasmas will generally charge negatively with respect to the plasma potential. Here we analyze in some level of detail the structure of the non-neutral layer adjacent to materials in contact with a plasma. To simplify our analysis, we assume the material is electrically isolated and catalytic, and we make use of a 1D model.

The region adjacent to the material wall is schematically shown below. We emphasize the distinction in the potential structure between the quasi-neutral plasma, a quasi-neutral collisional pre-sheath and the non-neutral sheath.



Very quickly, on the order of the inverse of the plasma frequency, the potential and number distributions reach an equilibrium as the current densities of electrons and ions become equal. All ions directed towards the wall eventually fall on it, while electrons of lower energy than $e\phi_w$ are reflected. The distribution function of ions is therefore far from isotropic in the sheath, while electrons can still be considered to be near-Maxwellian. From here, we write the current densities for ions and electrons respectively as,

$$j_i = en_i v_i \qquad \text{and} \qquad j_e = -e\frac{n_{ew}\bar{c}_e}{4} \tag{1}$$

where n_{ew} is the electron number density adjacent to the wall and their mean thermal speed is,

$$\bar{c}_e = \sqrt{\frac{8kT_e}{\pi m_e}}$$

Since the electron population is assumed Maxwellian, we use the Boltzmann equilibrium relation,

$$n_{ew} \;=\; n_{e_\infty} \exp\left(-\frac{e\phi_w}{kT_e}\right) \tag{2}$$

where n_{e_∞} is the plasma density (far away from the wall).

To determine the potential structure we first need to find what the ion flux is. To do this, we write the equations of motion of ions and electrons in the collisional pre-sheath,

$$
\begin{aligned}
m_i n_i v_i \frac{dv_i}{dx} + \frac{dp_i}{dx} &= e n_i E - F_i \\
m_e n_e v_e \frac{dv_e}{dx} + \frac{dp_e}{dx} &= -e n_e E - F_e
\end{aligned}
\tag{3}
$$

where F_i and F_e are frictional forces (per unit volume) on ions and electrons due to collisions in the pre-sheath. We add these equations and neglect electron inertia,

$$
m_i n_i v_i \frac{dv_i}{dx} + \frac{d(p_i + p_e)}{dx} = -F \tag{4}
$$

where we have made use of the quasi-neutrality assumption in the pre-sheath and we lumped together drag forces into F. Since the ion flux is conserved $\Gamma_i = n_i v_i$, we write,

$$
\frac{dG}{dx} = -F \qquad \text{where} \qquad G = m_i v_i \Gamma_i + \frac{\Gamma_i}{v_i} k T_e \tag{5}
$$

in which, in addition to quasi-neutrality, we used $p = nkT$ and assumed $T_e \gg T_i$.

We immediately notice that as we approach the wall, due to collisions, that the quantity G decreases in magnitude until reaching a minimum. This is accomplished by increasing the ion velocity to,

$$
v_{i,minG} = v_B = \sqrt{\frac{kT_e}{m_i}} \tag{6}
$$

This counter-intuitive result indicates that frictional forces accelerate ions to the terminal velocity v_B, called the *Bohm* velocity, or the ambipolar speed of sound.

This is the velocity at which ions enter the plasma sheath after falling through a small, but not-negligible potential drop in the pre-sheath. Notice that this peculiar velocity uses the temperature of the electrons, but the ion mass.

Going back to Eq. (1), we can evaluate the ion current density at the sheath boundary, where $n_i = n_e$ and $v_i = v_B$. The Boltzmann relation Eq. (2) is used once more but now to relate the plasma density at the pre-sheath boundary, to the bulk plasma density,

$$
n_i = n_e = n_{e_\infty} \exp\left(-\frac{e\phi_{ps}}{kT_e}\right) = n_{e_\infty} \exp\left(-\frac{m_i v_B^2}{2kT_e}\right) = n_{e_\infty} e^{-1/2} \tag{7}
$$

The current density balance then reads,

$$
e n_{e_\infty} e^{-1/2} \sqrt{\frac{kT_e}{m_i}} = e \frac{n_{e_\infty}}{4} \sqrt{\frac{8kT_e}{\pi m_e}} \exp\left(-\frac{e\phi_w}{kT_e}\right) \tag{8}
$$

We then find that the sheath potential can be approximated by,

$$\phi_w \approx \frac{kT_e}{e} \ln \sqrt{\frac{m_i}{m_e}} \tag{9}$$

For instance, for a Xe plasma with an electron population with 3 eV, the wall potential will be biased to about -18 V.