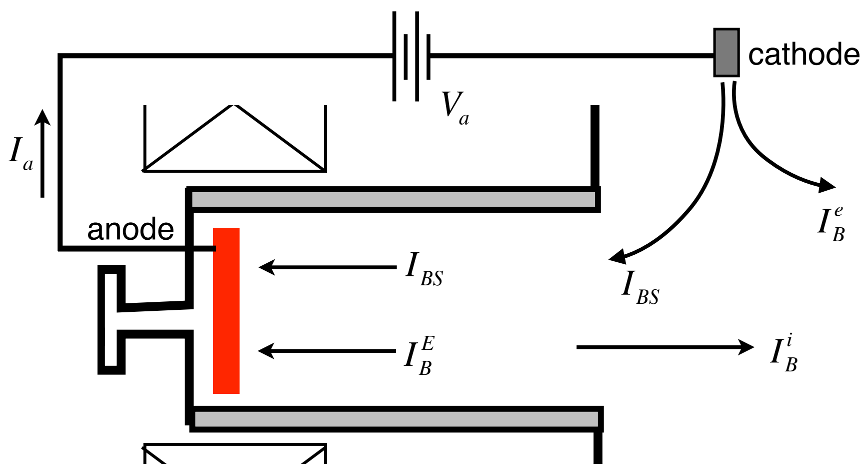
Lecture 19: Hall Thruster Efficiency

For a given mass flow $\dot{m}$ and thrust F, we would like to minimize the running power P. Define a thruster efficiency,

$$\eta = \frac{F^2}{2\dot{m}P} \tag{1}$$

where $F^2/2\dot{m}$ is the minimum required power. The actual power is,

$$P = I_a V_a \tag{2}$$



Where V_a is the accelerating voltage and I_a the current through the power supply (or anode current, or also cathode current). Of the I_a current of electrons injected by the cathode, a part I_B goes to neutralize the beam, and the rest, I_{BS} back-streams into the thruster.

Since no net current is lost to the walls,

$$I_a = I_B + I_{BS} \tag{3}$$

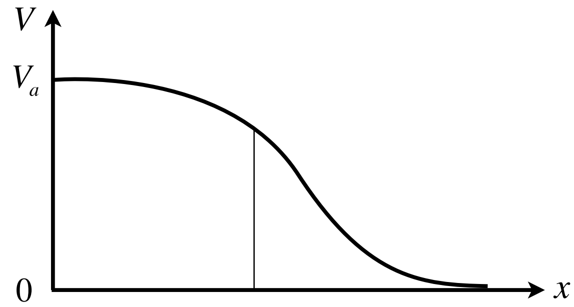
The thrust is due to the accelerated ions only. These are created at locations along the thruster which have different potentials $V(x)$, and hence accelerate to different speeds. Then,

$$F = \int c \, d\dot{m}_i \tag{4}$$

where,

$$c = \sqrt{\frac{2eV}{m_i}} \tag{5}$$

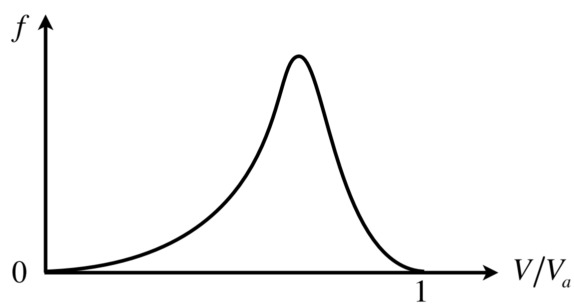
Suppose the part $d\dot{m}_i$ of $\dot{m}_i$ is created in the region where V decreases by dV, and define an ionization distribution function $f(V)$ by,



$$\frac{d\dot{m}_i}{\dot{m}_i} \;=\; -f\left(\frac{V}{V_a}\right)\frac{dV}{V_a} \tag{6}$$

or, with $\phi = V/V_a$,

$$\frac{d\dot{m}_i}{\dot{m}_i} \;=\; -f\left(\phi\right)d\phi$$



From the definition, $f(\phi)$ satisfies,

$$\int_0^1 f(\phi)\; d\phi \;=\; 1 \tag{7}$$

Then, from Eq. (4-5),

$$F \;=\; -\dot{m}_i\sqrt{\frac{2eV_a}{m_i}}\int_0^1 \sqrt{\phi}f(\phi)\; d\phi \tag{8}$$

and hence the efficiency is,

$$\eta \;=\; \frac{\dot{m}_i^2\dfrac{2eV_a}{m_i}\left(\displaystyle\int_0^1 \sqrt{\phi}f(\phi)\; d\phi\right)^2}{2\dot{m}I_aV_a} \tag{9}$$

Notice that the beam current I_B is related to $\dot{m}_i$ by $I_B = \dot{m}_i(e/m_i)$. We can therefore rewrite Eq. (9) as,

$$\eta \;=\; \left(\frac{\dot{m}_i}{\dot{m}}\right)\left(\frac{I_B}{I_a}\right)\left(\int_0^1 \sqrt{\phi}f(\phi)\; d\phi\right)^2 \tag{10}$$

where each of the factors is less than unity and can be assigned a separate meaning:

- $\eta_u = \frac{\dot{m}_i}{\dot{m}}$ is the utilization factor, i.e., it penalizes neutral gas flow.
- $\eta_a = \frac{I_B}{I_a}$ is the backstreaming, or anode efficiency, and penalizes electron leaking.
- $\eta_E = \left(\int_0^1 \sqrt{\phi} f(\phi) \, d\phi \right)^2$ is the non-uniformity, or energy efficiency.

It is clear that, since $\int_0^1 f(\phi) \, d\phi = 1$, we want to put most of $f(\phi)$ where ϕ is greatest, namely, we want to produce most of the ionization near the inlet. In that case $f(\phi) = \delta(\phi - 1)$, and $\eta_E = 1$. A somewhat pessimistic scenario would be $f(\phi) = 1$, namely $\frac{d\dot{m}_i}{dx}$ proportional to $-\frac{dV}{dx}$, i.e., ionization rate proportional to field strength. In that case,

$$\eta_E = \left(\int_0^1 \sqrt{\phi} \, d\phi \right)^2 = \frac{4}{9}$$

Measurements [1] tend to indicate η_E is between 0.6 and 0.9, which means that ionization tends to occur early in the channel. This is to be expected, because that is where the backstreaming electrons have had the most chance to gain energy by *falling up* the potential.

The factor $\eta_u = \frac{\dot{m}_i}{\dot{m}}$ is related to the ionization fraction. Since,

$$\dot{m} = \dot{m}_i + \dot{m}_n = n_e c_i A + n_n c_n A$$

we have,

$$\eta_u = \frac{n_e c_i}{n_e c_i + n_n c_n} = \frac{(n_e/n_n)(c_i/c_n)}{1 + (n_e/n_n)(c_i/c_n)} \tag{11}$$

Since c_i/c_n is large ($c_n \sim$ neutral speed of sound, i.e., a few hundred m/s, while $c_i \sim gI_{sp} \sim$ 20,000 m/s), η_u can be high even with n_e/n_n no more than a few percent. Data [1] show η_u ranging from 40% to 90%.

The factor $\eta_a = I_B/I_a$ requires some discussion. Most of the ionization is due to the backstreaming electrons, so that we are not really free to drive I_B towards I_a since $I_{BS} = I_a - I_B$. What we need to strive for is,

(a) Conditions which favor creation of as many ions as possible per backstreaming electron.

(b) Minimization of ion-electron losses to the walls, once they are created.

This can be quantified as follows: Let β be the number of secondary electrons (and of ions) produced per backstreaming electron, and let α be the fraction of these new $e - i$ pairs which is lost by recombination on the walls. Then, per backstreaming electron, $(1 - \alpha)\beta$

ions make it to the beam, and an equal number of cathode electrons are used to neutralize them. Therefore,

$$\eta_a = \frac{I_B}{I_a} = \frac{(1-\alpha)\beta}{1+(1-\alpha)\beta} \tag{12}$$

Clearly, we want $\beta \gg 1$ and $\alpha \ll 1$. The first ($\beta \gg 1$) implies lengthening the electron path by means of the applied radial magnetic field, and also using accelerating potentials which are not too far from the range of energies where ionization is most efficient (typically 30-80 Volts, a few times eV_i). This last condition creates some difficulties with heavy ions, which require higher accelerating potentials for a given exit speed.

The condition $\alpha \ll 1$ implies minimization of insulation surfaces on which the recombination can take place, and arrangement of the electric fields such that ions are not directly accelerated into walls. This is difficult to achieve without detailed surveys of equipotential surfaces.

References

[1] Komurasaki, K, Hirakowa, M. and Arakawa, Y., IEPC paper 91-078. 22nd Electric Propulsion Conference, Viareggio, Italy, Oct. 1991.